ENERGY APPROACH TO SOLVING PROBLEMS OF NONCLASSICAL THEORY FOR NONUNIFORM ANISOTROPIC SHELLS CONTAINING A CRACK-SLIT

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An energy approach is suggested in the present work for solving linear and nonlinear boundary problems of nonclassical theory for piecewise-inhomogeneous (layered) anisotropic shells containing defects of the crack type. Layered shells are considered assembled symmetrically in a geometrical and physical sense in relation to their coordinate surface containing one or two colinear through crack-slits rectilinear in plan. Layers exhibit elastic properties of either an isotropic, or a transversely isotropic, or an orthotropic uniform material. The number of layers n in a package may be either even or uneven. At the separation surfaces of layers contact conditions are fulfilled, i.e., there is absence of sliding and separation between layers [1-4]. The behavior of the shells in question with cracks is described by a geometrically nonlinear theory of the Timoshenko type.

1. Statement of Problem. We formulate a variation problem of geometrically nonlinear Timoshenko type theory for a layered orthogropic flattened shell with a crack length 2L arranged in the line of least resistance. We relate the coordinate surface of the shell with the crack to a Cartesian rectangular coordinate system X, Y, and Z whose origin is at the center of the crack and its axis coincides with lines of intersection for planes of elastic symmetry of all layers. A normal uniformly distributed load with intensity q_m operates at the surface of a shell with a slit.

The variation problem is set up as follows [5]. To determine the minimum value of functional V expressing the potential energy of a shell with a slit by proceeding from the condition that the first variation $\delta V = 0$ over the whole independent functional argument satisfying conditions at the shell contour [1-3, 6, 7]:

$$u_n = u_n^0, v_n = v_n^0, w_n = w_n^0, \varphi_n = \varphi_n^0, \psi_n = \psi_n^0,$$
$$N_n = N_n^0, S_n = S_n^0, Q_n = Q_n^0, M_n = M_n^0, H_n = H_n^0,$$

where the first part is kinematic and the second is static boundary conditions for the whole package; there are boundary conditions at the separation surface $|x| \le L$ with $y = 0$:

$$N_n = 0,\ S_n = 0,\ Q_n = 0,\ M_n = 0,\ H_n = 0.$$

The general solution for the linear problem for the shell in question with a slit is presented in the form of the sum of solutions: the basic solution (shell without a slit) and disturbed solution (shell with a slit). The variation problem for a disturbed shell is formulated as follows: to determine the minimum value of functional V which expresses the potential energy of a shell with a slit; independent functional arguments satisfying boundary conditions at the crack-slit surface:

$$N_n = N_n^1, S_n = S_n^1, Q_n = Q_n^1, M_n = M_n^1, H_n = H_n^1$$

($N_n^1, S_n^1, Q_n^1, M_n^1$ and H_n^1 are tensile, shear, and transverse forces, bending and torsional moments for the whole package taken from solution of the basic problem in that area where occurrence of a crack is assumed), and also conditions for "infinity"

$$u_n|_{\rho\to\infty} \to 0,\ v_n|_{\rho\to\infty} \to 0,$$
$$w_n|_{\rho\to\infty} \to 0,\ \varphi_n|_{\rho\to\infty} \to 0,\ \psi_n|_{\rho\to\infty} \to 0$$

(u_n, v_n, and w_n are displacements of the coordinate surface along axes X, Y, and Z respec-

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TABLE 1

Theory	h/L	E_x/E_y					
		2,0	4,0	8,0	16,0	32,0	60,0
Linear			*13* nodes				
	1	0.296	0,290	0,292	0,298	0,296	0,296
			25 nodes				
	1	0,315	0,318	0.317	0,318	0,318	0,319
Nonlinear			*13* nodes				
	1	0,305	0,306	0,304	0,308	0,307	0,307
			25 nodes				
	1	0,326	0,327	0,325	0,327	0,328	0,328

tively, φ_n, and ψ_n are angles of rotation for the normal and planes XZ and YZ, and ρ is the distance from the crack tip).

Currently solution of both the linear and nonlinear basic problems does not present any special difficulties [1-3, 6, 7]. Therefore we occupy ourselves with solving the disturbed problem.

2. Method of Solution. The amplitude of the local symmetrical disturbed condition for a multilayered orthotropic shell with a crack-slit is determined by the value of stress intensity factor K_I^0 which characterizes normal separation [4, 5].

In order to find factor K_I^0 for both linear and nonlinear problems an energy approach is suggested:

$$V = \frac{1}{2}\iint_S (\mathbf{q}\mathbf{r} + \mathbf{p}\boldsymbol{\gamma})(1 \pm k_1 h)(1 \pm k_2 h)\,dS,$$
$$G_I^0 = \partial V/\partial L,\ (K_I^0)^2 = G_I^0 \Big/ \left\{\left(\frac{a_{11}a_{22}}{2}\right)^{1/2}\left[\left(\frac{a_{22}}{a_{11}}\right)^{1/2} + \frac{2a_{12}+a_{66}}{2a_{11}}\right]^{1/2}\right\}. \tag{2.1}$$

Here V is potential energy of a multilayer linear and nonlinear shell with a slit which may be determined by the Clapeyron theorem; **q**, **p** are vectors of external loads: forces and moments; **r**, $\boldsymbol{\gamma}$ are vectors of displacements and rotation angles for points of the central surface; h is shell thickness; k_1, k_2 are normal curvatures of the central surface of the shell; S is shell surface; G_I^0 is intensity of energy released with an increase crack surface by hdL; $K_I^0 = K_I + K_I^*$ is total stress intensity factor; K_I is stress intensity factor in tension, K_I^* is the same with bending; $a_{11} = \sum_{i=1}^{n} a_{11}^i;\ a_{11}^i = 1/E_x^i;\ a_{22} = \sum_{i=1}^{n} a_{22}^i;\ a_{22}^i = 1/E_y^i;\ a_{12} = \sum_{i=1}^{n} a_{12}^i;\ a_{12}^i = -\nu_{xy}^i/E_x^i;\ a_{66} = \sum_{i=1}^{n} a_{66}^i;\ a_{66}^i = 1/\mu_{xy}^i$; E_x^i, E_y^i are Young's moduli for the layers; μ_{xy}^i is Shear modulus for the layers; ν_{xy}^i is Poisson's ratio for the layers.

In calculating intensities G_I^0 we shall use two methods: first the derivative is replaced by finite differences, and here it is necessary as a minimum to calculate for two crack lengths, it is a simple version of the energy approach to determining K_I^0 and it is called the method of differential stiffness (compliance method); second in order to obtain the increment in potential energy for a shell with a crack

$$\Delta V = V(L + \Delta L) - V(L) \tag{2.2}$$

It is not possible to consider the increase in crack length for one of several cells of the grid, and due to this the change in coordinates of the crack tip within a cell to prescribe its propagation. As a result of this observation there is a change in the stiffness of elements immediately adjacent to the crack tip, and by taking this into account sets of linear

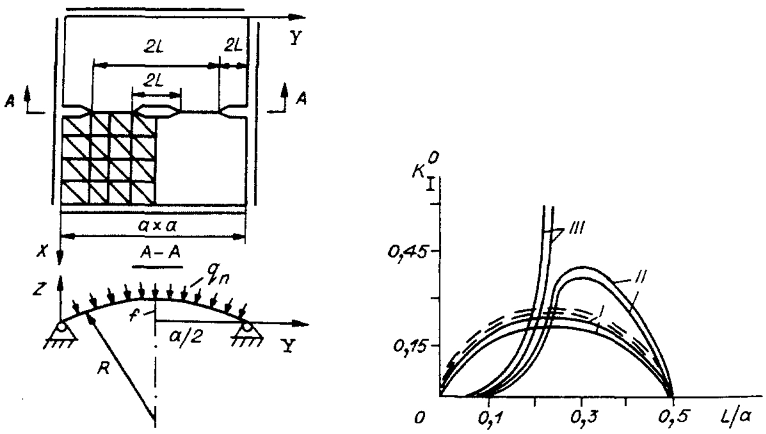

Fig. 1

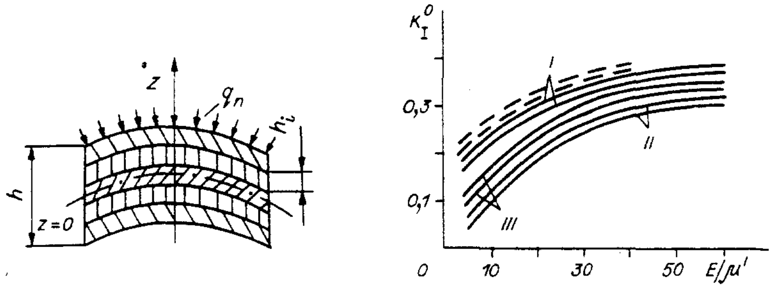

Fig. 2

and nonlinear equations are solved. A subprogram is written which makes it possible to realize this feature and thereby to calculate for one solution the change in energy by Eq. (2.2). This version of the energy approach is called the method of virtual crack growth.

In view of the fact that for linear problems the Timoshenko model used in the present work makes it possible to combine stress intensity factors in tension and bending, derivation of the relationship between factor K_I^0 and intensity G_I^0 from (2.1) involves the following: it takes the hypothesis that the surface layer of the next shell layer in the region of the crack tip locally at the stretched side behaves similar to a plate which is in a condition of uniform tension; then considering the principle of superposition displacement of the points at the shell surface layer in question is presented in the form of sum $v = v_1 + v_2$ (v_1 is displacement of a point with coordinate $z = -h/2$ in tension, v_2 is the same with bending). Assuming that the crack grows with constant stress σ_y arising at point $z = -h/2$, the jump in displacment v at the crack gives the flow of energy sought with intensity

$$G_I^0 = \frac{\partial V}{\partial L} = \frac{\partial}{\partial L}\left(\frac{1}{2}\int\limits_{-h/2}^{h/2}\int\limits_{-L}^{L}\sigma_y(x,0)\left(v^+ - v^-\right)dx\,dz\right), \tag{2.3}$$

where $v^+ - v^- = 2a_{22}\sigma_y(L^2 - x^2)^{1/2}\,\mathrm{Re}\left[i\left(\frac{s_1 + s_2}{s_1 s_2}\right)\right]$ is size of the jump; $|x| \leqslant L$; s_t $(t = 1, 2)$ are roots of the biquadratic equation for orthotropic material.

By substituting an expression for the jump in (2.3), integrating with respect to h and L and differentiating with respect to L, and also considering that $K_I^0 = \sigma_y\sqrt{\pi L}$ and $h = 1$, we have

$$G_I^0 = (K_I^0)^2 \frac{a_{22}}{2} \operatorname{Re}\left[i \frac{s_1 - s_2}{s_1 s_2} \right]. \quad (2.4)$$

with values of s_t taken from [8], in (2.4) we obtain the relationship sought, and it is possible to use it for nonlinear problems based on a theorem proved in [9] from which it follows that the main part of the asymptotics for the energy solution at the slit tip is governed by its linear part. If the approximate nature of the grounds for using the dependence of K_I^0 on G_I^0 for the nonlinear case does not satisfy the reader, then in the calculation it is possible to limit oneself to calculating intensity G_I^0.

If in the third equation of (2.1) elastic constants are substituted which characterize transversely isotropic layers, then we find an expression $(K_I^0)^2 = G_I^0 E$, where $E = \frac{1}{h}\sum_{i=1}^{n} h_i E_i$ is the elastiscity modulus for the package. Relationship (2.3) will be used in calculating the factor for a layered transversely isotropic and isotropic shells with cracks.

The main difficulty in both methods is calculation of the component vectors at displacements and rotation angles. The finite element method (FEM) is used in a displacement version. The main part of the FEM is derivation of the stiffness matrix by means of which it is possible to establish the relationship between nodal forces and displacements corresponding to them. On the basis of [4, 10] a stiffness matrix is obtained for a rectangular finite element of nonzero Gaussian curvature of layered orthotropic material for geometric nonlinear theory of the Timoshenko type, and it is also used for calculating layered transversely isotopic and isotropic arbitrary shells with cracks.

3. Numerical Examples and Analysis. We consider a five-layer isotropic, square, freely supported cylindrical panel with three cases of crack arrangement; I) in the center, II) at the side, III) at both sides. Geometrical and physical characteristics of the panel shown in Fig. 1 are as follows: h = 0.01 m, R = 0.20 m, a = 0.30 m; f = 0.0677 m, ν = 0.3. Presented in Fig. 1 are curves for the dependence of K_I^0 on parameter L/a. For all situations of crack arrangement the axis of panel symmetry was broken down into 13 and 25 nodes. With breakdown of the panel axis into 13 nodes factor K_I^0 was calculated by the compliance method, and with breaking it down into 25 nodes it was determined by the method of virtual crack growth (a feature of the methods was considered; for the first the simplicity and availability, and for the second accuracy and economy). The relative error between the curves obtained with breaking down the axis of symmetry into 13 and 25 nodes with all cases of slit arrangement did not exceed 6%.

For the panel described above, but made of transversely isotropic layers, curves are plotted in Fig. 2 for the dependence of $K_I{}^0$ on μ' (μ' is a parameter which characterizes the transversality of the panel over the thickness) for all situations of crack arrangement with h/L = 1. The method of breaking down the axis of panel symmetry and methods for obtaining values of K_I^0 are similar to the previous case. It can be seen from Fig. 2 that factor K_I^0 depends markedly on shell transversality over the thickness.

We consider precisely the same panel as that above, but made of orthotropic material (glass-reinforced plastic) with a crack at the center, and we determine the effect of degree of orthotropy E_x/E_y on $K_I{}^0$. Results are presented in Table 1 from the analysis of which it can be seen that the degree of orthotropy does not affect $K_I{}^0$. The relative error between calculations relates to the calculation error and the effect of grid spacing.

Shown in Figs. 1 and 2 by broken lines are curves obtained on the basis of geometrical nonlinear theory, and by analyzing them it is possible to conclude the geometric nonlinearity affects the amplitude of factor K_I^0, i.e., with use of more precise shell theory it becomes softer and the value of the intensity factor increases.

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